

Spin Light of Neutrino in Dense Matter

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Abstract

We develop the quantum theory of the spin light of neutrino ($SL\nu$) exactly accounting for the effect of background matter. Contrary to the already performed studies of the $SL\nu$, in this paper we derive explicit and closed expressions for the $SL\nu$ rate and power and for the emitted photon energy, which are valid for an arbitrary matter density (including very high values). The spatial distribution of the radiation power and the dependence of the emitted photon energy on the direction of radiation are also studied in detail for the first time. We analyze the $SL\nu$ polarization properties and show that within a wide range of neutrino momenta and matter densities the $SL\nu$ radiation is circularly polarized. Conditions for effective $SL\nu$ photon propagation in the electron plasma are discussed. It is also shown that in dense matter the average energy of the emitted photon can reach values in the range from one third of the neutrino momentum up to one half of the neutrino energy in matter. The main features of the studied radiation are summarized, and possibilities for the $SL\nu$ production during different astrophysical and cosmology processes are discussed.

1 Spin light of a neutrino in matter

There exist various mechanisms for the production of electromagnetic radiation by a massive neutrino moving in a background environment (see, for instance, [1])¹. We have

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¹A brief classification of the known mechanisms of the electromagnetic radiation by a neutrino is given in the first paper of [2].

recently shown [2] within the quasi-classical approach, that a massive neutrino moving in background matter can emit a new type of electromagnetic radiation. This radiation has been termed the "spin light of neutrino" ($SL\nu$) in matter. In [3] we have also considered $SL\nu$ in gravitational fields of rotating astrophysical objects. Developing the quantum theory of this phenomenon [4, 5], we have demonstrated that $SL\nu$ arises owing to two underlying phenomena: (i) the shift of neutrino energy levels in matter, that are different for the two opposite neutrino helicity states, and (ii) the emission of an $SL\nu$ photon in the process of neutrino transition from the "excited" helicity state to the low-lying helicity state in matter. However, calculations of the transition rate and radiation power have been performed in the limit of a low matter density and, therefore, evaluation of a consistent quantum theory of $SL\nu$ still remains an open issue.

In this paper we develop the quantum theory of $SL\nu$, exactly taking into account the effect of background matter, and obtain expressions for the $SL\nu$ rate and power that are valid for any value of the matter density parameter (see also [6]). In Section 2 we briefly discuss the modified Dirac equation and the neutrino energy spectrum in the presence of matter which are then used (Section 3) for derivation of the $SL\nu$ transition rate and power. We get an exact expression for the emitted photon energy as a function of the initial neutrino energy and the matter density parameter. The dependence of the photon energy on the direction of the photon propagation is analyzed, and a detailed study of the radiation spatial distribution is also performed. We also derive the exact and closed expressions for the rate and total radiation power of $SL\nu$ and analyze them for different limiting cases. The $SL\nu$ polarization properties are studied in Section 4, and the conclusion is made concerning the total circular polarization of the emitted photons. Section 5 is devoted to the discussion of restrictions on the propagation of $SL\nu$ photons that can be set by the electron plasma. In conclusion (Section 6) we give a summary of the investigated properties of $SL\nu$ in matter. The $SL\nu$ production during processes of collapse and coalescence of neutron stars or a neutron star being "eaten up" by the black hole at the center of our Galaxy are also discussed as one of possible mechanisms of gamma-rays production.

2 The modified Dirac equation in matter

To account for the influence of background matter on neutrinos we use the approach [4] (similar to the Furry representation in quantum electrodynamics) that is based on the exact solutions of the modified Dirac equation for a neutrino in matter:

$$\left\{ i\gamma_\mu \partial^\mu - \frac{1}{2}\gamma_\mu(1 + \gamma_5)f^\mu - m \right\} \Psi(x) = 0. \quad (1)$$

In the case of matter composed of electrons

$$f^\mu = \frac{G_F}{\sqrt{2}}(1 + 4 \sin^2 \theta_W)j^\mu, \quad (2)$$

where the electron current j^μ is given by

$$j^\mu = (n, n\mathbf{v}). \quad (3)$$

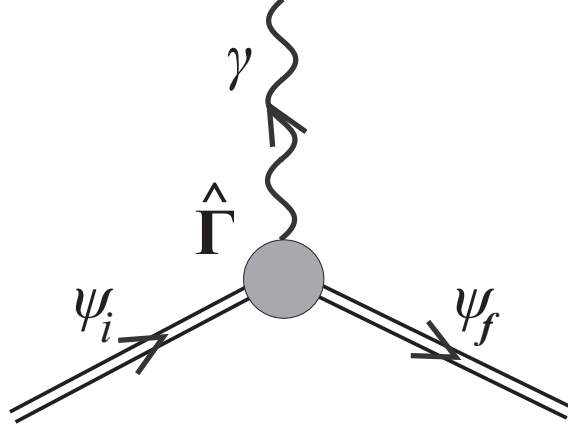


Figure 1: The effective diagram of the $SL\nu$ photon emission process. The broad lines correspond to the initial and final neutrino states in the background matter.

Here θ_W , n and \mathbf{v} are, respectively, the Weinberg angle, the number density of background electrons and the speed of the reference frame in which the mean momentum of the electrons is zero. As it has been shown [4] the solutions of Eq.(1) are given by

$$\Psi_{\varepsilon, \mathbf{p}, s}(\mathbf{r}, t) = \frac{e^{-i(E_\varepsilon t - \mathbf{p}\mathbf{r})}}{2L^{\frac{3}{2}}} \begin{pmatrix} \sqrt{1 + \frac{m}{E_\varepsilon - \alpha m}} \sqrt{1 + s \frac{p_3}{p}} \\ s \sqrt{1 + \frac{m}{E_\varepsilon - \alpha m}} \sqrt{1 - s \frac{p_3}{p}} e^{i\delta} \\ s\varepsilon \sqrt{1 - \frac{m}{E_\varepsilon - \alpha m}} \sqrt{1 + s \frac{p_3}{p}} \\ \varepsilon \sqrt{1 - \frac{m}{E_\varepsilon - \alpha m}} \sqrt{1 - s \frac{p_3}{p}} e^{i\delta} \end{pmatrix}, \quad (4)$$

where energy spectrum is

$$E_\varepsilon = \varepsilon \sqrt{\mathbf{p}^2 \left(1 - s\alpha \frac{m}{p}\right)^2 + m^2 + \alpha m}, \quad (5)$$

$$\alpha = \frac{1}{2\sqrt{2}} \tilde{G}_F \frac{n}{m}, \quad \tilde{G}_F = G_F(1 + 4\sin^2 \theta_W). \quad (6)$$

In equations (4)-(6) m , \mathbf{p} and $s = \pm 1$ are the neutrino mass, momentum and helicity, respectively. The quantity $\varepsilon = \pm 1$ splits the solutions into two branches that in the limit of vanishing matter density, $\alpha \rightarrow 0$, reproduce the positive and negative-frequency solutions for the Dirac equation in vacuum.

Note that generalization to the case of matter composed of different types of fermions is straightforward [4], and the correct value for the neutrino energy difference corresponding to the Mikheyev-Smirnov-Wolfenstein effect [7] can be recovered from (5). The modified effective Dirac equations for a neutrino interacting with various background environments within different models were previously used [8] in a study of the neutrino dispersion relations, neutrino mass generation and for derivation of the neutrino oscillation probabilities

in matter. On the same basis, the neutrino decay into an antineutrino and a light scalar particle (majoron), as well as the corresponding process of the majoron decay into two neutrinos or antineutrinos, were studied in the presence of matter [9].

3 The $SL\nu$ transition rate and power

The $SL\nu$ amplitude calculated within the developed quantum theory is given by (see also [4])

$$S_{fi} = -\mu\sqrt{4\pi} \int d^4x \bar{\psi}_f(x)(\hat{\Gamma}\mathbf{e}^*) \frac{e^{ikx}}{\sqrt{2\omega L^3}} \psi_i(x), \quad (7)$$

$$\hat{\Gamma} = i\omega \{ [\boldsymbol{\Sigma} \times \boldsymbol{\varkappa}] + i\gamma^5 \boldsymbol{\Sigma} \},$$

where μ is the neutrino magnetic moment, ψ_i and ψ_f are the exact solutions of equation (1) for the initial and final neutrino states [4], $k^\mu = (\omega, \mathbf{k})$ and \mathbf{e}^* are the photon momentum and polarization vector, $\boldsymbol{\varkappa} = \mathbf{k}/\omega$ is the unit vector pointing in the direction of the emitted photon propagation.

Integration over time in (7) yields

$$S_{fi} = -\mu\sqrt{\frac{2\pi}{\omega L^3}} 2\pi\delta(E' - E + \omega) \int d^3x \bar{\psi}_f(\mathbf{r})(\hat{\Gamma}\mathbf{e}^*) e^{i\mathbf{k}\mathbf{r}} \psi_i(\mathbf{r}), \quad (8)$$

where the delta-function stands for energy conservation, E and E' are the energies of the initial and final neutrino states in matter. Performing integration over the spatial coordinates, we can recover the delta-functions for the three components of the momentum. Finally, we get the law of energy-momentum conservation for the considered process,

$$E = E' + \omega, \quad \mathbf{p} = \mathbf{p}' + \boldsymbol{\varkappa}, \quad (9)$$

where \mathbf{p} and \mathbf{p}' are the initial and final neutrino momenta, respectively. From (9) it follows that the emitted photon energy ω exhibits a critical dependence on the helicities of the initial and final neutrino states. In the case of electron neutrino moving in matter composed of electrons α is positive. It follows that $SL\nu$ can arise only when the neutrino initial and final states are characterized by $s_i = -1$ and $s_f = +1$, respectively. One can also conclude that in the process considered the relativistic left-handed neutrino is converted to the right-handed neutrino. A discussion of the main properties of $SL\nu$ emitted by different flavor neutrinos moving in matter composed of electrons, protons and neutrons can be found in [4] (see also [?]).

The emitted photon energy in the considered case ($s_i = -s_f = -1$), obtained as an exact solution of equations (9), is

$$\omega = \frac{2\alpha m p [(E - \alpha m) - (p + \alpha m) \cos \theta]}{(E - \alpha m - p \cos \theta)^2 - (\alpha m)^2}, \quad (10)$$

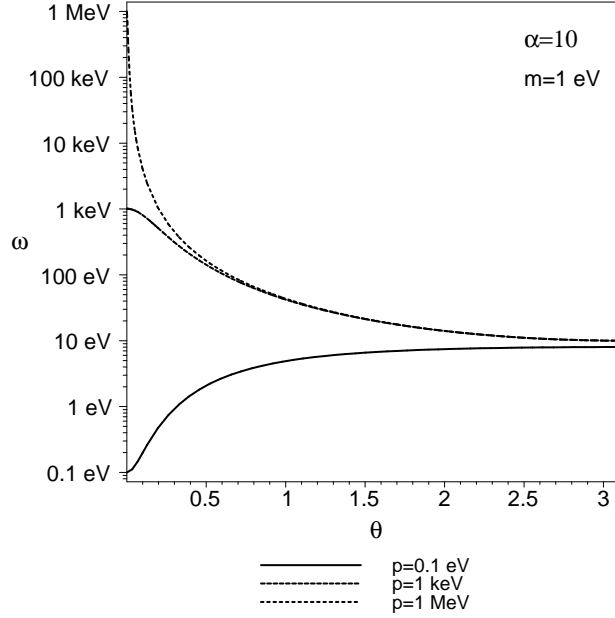


Figure 2: Angular dependence of photon energy for different values of the neutrino momentum: 1) the solid line corresponds to $p = 0.1 \text{ eV}$, 2) the dashed-solid line corresponds to $p = 1 \text{ keV}$ and 3) the dashed line corresponds to $p = 1 \text{ MeV}$. The matter density parameter is $\alpha = 10$.

where θ is the angle between \mathbf{z} and the direction of the initial neutrino propagation. The photon energy is a rather complicated function of the neutrino energy E and momentum p , the matter density parameter α and the angle θ . Fig.1 shows the angular dependence of the photon energy for different values of the neutrino momentum. From this figure one may expect that in the case of relativistic neutrinos ($p \gg m$) and not very dense matter ($\alpha \ll \frac{p}{m}$) $SL\nu$ is collimated along the direction of the neutrino momentum p (see the dashed and solid-dashed curves). On the contrary, in the case of non-relativistic neutrinos ($p \ll m$) and $\alpha \gg \frac{p}{m}$ (see solid line in Fig.1) the emitted photon energy in the direction of the neutrino momentum p is suppressed. It should also be noted that for all cases shown in Fig.1 the energies ω of the photons radiated at large angles θ are of the order of $\sim \alpha m$. In the case of a not very high density of matter, when the parameter $\alpha \ll 1$, one can expand the photon energy (10) over α and in the linear approximation get the result of [4, 5]:

$$\omega = \frac{1}{1 - \beta \cos \theta} \omega_0, \quad (11)$$

where

$$\omega_0 = \frac{\tilde{G}_F}{\sqrt{2}} n \beta, \quad \beta = \frac{p}{\sqrt{p^2 + m^2}}. \quad (12)$$

Using the expressions for the amplitude (8) and for the photon energy (10) we calculate the spin light transition rate and total radiation power exactly accounting for the matter

density parameter:

$$\Gamma = \mu^2 \int_0^\pi \frac{\omega^3}{1 + \tilde{\beta}' y} S \sin \theta d\theta, \quad (13)$$

$$I = \mu^2 \int_0^\pi \frac{\omega^4}{1 + \tilde{\beta}' y} S \sin \theta d\theta, \quad (14)$$

where

$$S = (\tilde{\beta}\tilde{\beta}' + 1)(1 - y \cos \theta) - (\tilde{\beta} + \tilde{\beta}')(\cos \theta - y). \quad (15)$$

Here we introduce the notations

$$\tilde{\beta} = \frac{p + \alpha m}{E - \alpha m}, \quad \tilde{\beta}' = \frac{p' - \alpha m}{E' - \alpha m}, \quad (16)$$

where the final neutrino energy and momentum are, respectively,

$$E' = E - \omega, \quad p' = K\omega - p, \quad (17)$$

and

$$y = \frac{\omega - p \cos \theta}{p'}, \quad K = \frac{E - \alpha m - p \cos \theta}{\alpha m}. \quad (18)$$

Performing the integration in (13), we obtain for the $SL\nu$ rate in matter

$$\begin{aligned} \Gamma = & \frac{1}{2(E-p)^2(E+p-2\alpha m)^2(E-\alpha m)p^2} \\ & \times \left\{ (E^2 - p^2)^2 (p^2 - 6\alpha^2 m^2 + 6E\alpha m - 3E^2) ((E - 2\alpha m)^2 - p^2)^2 \right. \\ & \times \ln \left[\frac{(E+p)(E-p-2\alpha m)}{(E-p)(E+p-2\alpha m)} \right] + 4\alpha m p [16\alpha^5 m^5 E (3E^2 - 5p^2) \\ & - 8\alpha^4 m^4 (15E^4 - 24E^2 p^2 + p^4) + 4\alpha^3 m^3 E (33E^4 - 58E^2 p^2 + 17p^4) \\ & - 2\alpha^2 m^2 (39E^2 - p^2) (E^2 - p^2)^2 + 12\alpha m E (2E^2 - p^2) (E^2 - p^2)^2 \\ & \left. - (3E^2 - p^2) (E^2 - p^2)^3 \right] \Big\}, \end{aligned} \quad (19)$$

where the energy of the initial neutrino is given by (5) with $\varepsilon = -s_i = 1$.

As it follows from (19), the $SL\nu$ rate is a rather complicated function of neutrino momentum p and mass m , it also non-trivially depends on the matter density parameter α . In the limit of a low matter density, $\alpha \ll 1$, we get

$$\Gamma \simeq \frac{64}{3} \frac{\mu^2 \alpha^3 p^3 m}{E_0}, \quad (20)$$

where $E_0 = \sqrt{p^2 + m^2}$. The obtained expression is in agreement with our results of [2, 4, 5]. Note that the considered limit of $\alpha \ll 1$ can be appropriate even for a very dense media

of neutron stars with $n \sim 10^{33} \text{ cm}^{-3}$ because $\frac{1}{2\sqrt{2}}\tilde{G}_F n \sim 1 \text{ eV}$ for a medium characterized by $n = 10^{37} \text{ cm}^{-3}$.

Performing also the integration in (14), we obtain the total $SL\nu$ radiation power in matter

$$\begin{aligned}
I = & \frac{5}{2(E-p)^3(E+p-2\alpha m)^3 p^2} \times \left\{ (E+p)^2(E-m)^3(E+p-2\alpha m)^3 \right. \\
& \times (E-p-2\alpha m)^2 \left(2\alpha^2 m^2 - 2\alpha m(E + \frac{1}{5}p) + E^2 - \frac{3}{5}p^2 \right) \\
& \times \ln \left(\frac{(2\alpha m - p - E)(E-p)}{(2\alpha m + p - E)(E+p)} \right) \\
& - 4\alpha m p \left(32\alpha^6 m^6 \left(E^4 - pE^3 - \frac{5}{3}p^2 E^2 + \frac{5}{3}p^3 E + \frac{8}{15}p^4 \right) \right. \\
& - 96\alpha^5 m^5 \left(E^5 - \frac{23}{30}pE^4 - \frac{83}{45}p^2 E^3 + \frac{11}{9}p^3 E^2 + \frac{38}{45}p^4 E - \frac{1}{10}p^5 \right) \\
& + 128\alpha^4 m^4 \left(E^6 - \frac{47}{80}pE^5 - \frac{511}{240}p^2 E^4 + \frac{127}{120}p^3 E^3 + \frac{157}{120}p^4 E^2 - \frac{89}{240}p^5 E - \frac{7}{48}p^6 \right) \\
& - 96(E^2 - p^2)\alpha^3 m^3 \left(E^5 - \frac{53}{120}pE^4 - \frac{3}{2}p^2 E^3 + \frac{89}{180}p^3 E^2 + \frac{47}{90}p^4 E - \frac{19}{360}p^5 \right) \\
& + 42(E^2 - p^2)^2 \alpha^2 m^2 \left(E^4 - \frac{32}{105}pE^3 - \frac{314}{315}p^2 E^2 + \frac{4}{21}p^3 E + \frac{17}{105}p^3 \right) \\
& \left. - 10\alpha m(E^2 - p^2)^3 \left(E^3 - \frac{4}{25}pE^2 - \frac{17}{25}p^2 E + \frac{2}{25}p^3 \right) + (E^2 - p^2)^4 \left(E^2 - \frac{3}{5}p^2 \right) \right\}. \quad (21)
\end{aligned}$$

In the case $\alpha \ll 1$, we get

$$I \simeq \frac{128}{3} \mu^2 \alpha^4 p^4 \quad (22)$$

in agreement with Refs. [2, 4, 5].

Let us consider the $SL\nu$ rate and power for the different limiting values of the neutrino momentum p and matter density parameter α . In the relativistic case $p \gg m$ from (19) we get

$$\Gamma = \begin{cases} \frac{64}{3} \mu^2 \alpha^3 p^2 m, \\ 4 \mu^2 \alpha^2 m^2 p, \\ 4 \mu^2 \alpha^3 m^3, \end{cases} \quad I = \begin{cases} \frac{128}{3} \mu^2 \alpha^4 p^4, \\ \frac{4}{3} \mu^2 \alpha^2 m^2 p^2, \\ 4 \mu^2 \alpha^4 m^4, \end{cases} \quad \begin{array}{l} \text{for } \alpha \ll \frac{m}{p}, \\ \text{for } \frac{m}{p} \ll \alpha \ll \frac{p}{m}, \\ \text{for } \alpha \gg \frac{p}{m}, \end{array} \quad (23)$$

and in the opposite case, $p \ll m$, we have

$$\Gamma = \begin{cases} \frac{64}{3} \mu^2 \alpha^3 p^3, \\ \frac{512}{5} \mu^2 \alpha^6 p^3, \\ 4 \mu^2 \alpha^3 m^3, \end{cases} \quad I = \begin{cases} \frac{128}{3} \mu^2 \alpha^4 p^4, \\ \frac{1024}{3} \mu^2 \alpha^8 p^4, \\ 4 \mu^2 \alpha^4 m^4, \end{cases} \quad \begin{array}{l} \text{for } \alpha \ll 1, \\ \text{for } 1 \ll \alpha \ll \frac{m}{p}, \\ \text{for } \alpha \gg \frac{m}{p}. \end{array} \quad (24)$$

One can see that in the case of a very high matter density the rate and radiation power are determined by the background matter density only. Note that the obtained $SL\nu$ rate and radiation power for $p \gg m$ and $\alpha \gg \frac{m}{p}$ are in agreement with [?].

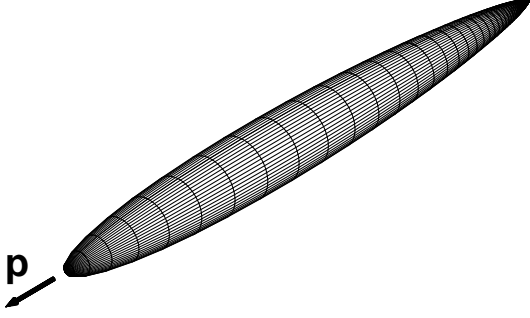


Figure 3: The spatial distribution of the $SL\nu$ radiation power for $p/m = 5$, $\alpha = 0.01$.

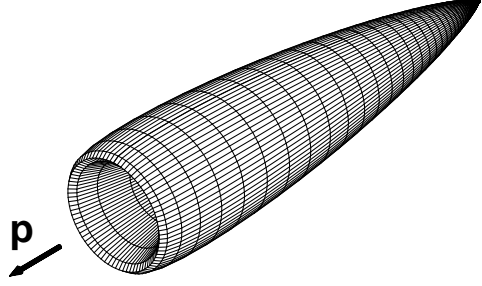


Figure 4: The spatial distribution of the $SL\nu$ radiation power for $p/m = 10^3$, $\alpha = 100$.

From the expressions for the $SL\nu$ rate and total power it is possible to get an estimate for the average emitted photon energy:

$$\langle\omega\rangle = \frac{I}{\Gamma}. \quad (25)$$

In the relativistic case, $p \gg m$, we get

$$\langle\omega\rangle \simeq \begin{cases} 2\alpha \frac{p^2}{m}, & \text{for } \alpha \ll \frac{m}{p}, \\ \frac{1}{3}p, & \text{for } \frac{m}{p} \ll \alpha \ll \frac{p}{m}, \\ \alpha m, & \text{for } \alpha \gg \frac{p}{m}. \end{cases} \quad (26)$$

For the matter parameter $\alpha \gg \frac{m}{p}$ we again confirm, here, the result obtained in [?]. In the non-relativistic case, $p \ll m$, we have for the average emitted photon energy

$$\langle\omega\rangle \simeq \begin{cases} 2\alpha p, & \text{for } \alpha \ll 1, \\ \frac{10}{3}\alpha^2 p, & \text{for } 1 \ll \alpha \ll \frac{m}{p}, \\ \alpha m, & \text{for } \alpha \gg \frac{m}{p}. \end{cases} \quad (27)$$

We should like to note that for a wide range of neutrino momenta p and density parameters α the $SL\nu$ power is collimated along the direction of the neutrino propagation. The shapes of the radiation power spatial distributions calculated with use of (14) in the case of $p > m$ for low and high matter density are shown in Figs.2 and 3, respectively. As it follows from these figures, the shape of the distribution depends on the density of matter. The shape of the spatial distribution of the radiation changes from projector-like to cap-like with increase of the matter density. From (14) it follows, that in the case of $p \gg m$ for a wide range of matter densities, $\alpha \ll \frac{p}{m}$, the direction of the maximum in the spatial distribution of the radiation power is characterized by the angle

$$\cos \theta_{max} \simeq 1 - \frac{2}{3}\alpha \frac{m}{p}. \quad (28)$$

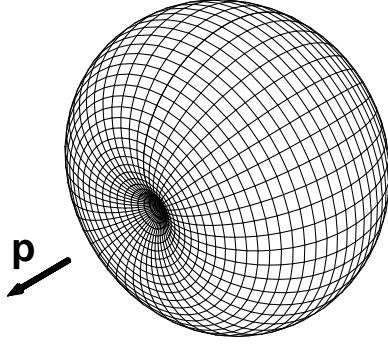


Figure 5: The spatial distribution of the $SL\nu$ radiation power for $p/m = 10$, $\alpha = 100$.

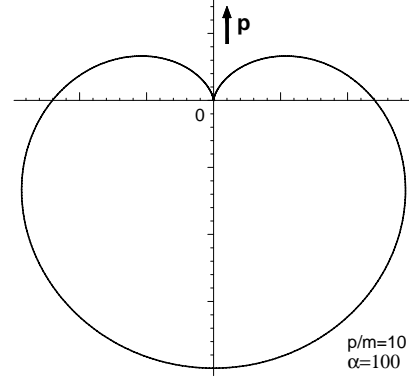


Figure 6: The two-dimensional cut of the spatial distribution of the $SL\nu$ radiation power shown in Fig.4, $p/m = 10$, $\alpha = 100$.

It follows that in a dense matter the $SL\nu$ radiation in the direction of the initial neutrino motion is strongly suppressed, whereas there is a luminous ring in the plane perpendicular to the neutrino motion. Note that the rate of the matter-induced neutrino majoron decay, as it was shown in the second paper of [9], exhibits a similar angular distribution.

From analysis of the spatial distribution of the $SL\nu$ radiation and the emitted photon average energy we predict an interesting new phenomenon that can appear if a bunch of neutrinos propagates in a very dense matter. In the case of relativistic neutrinos $p \gg m$ and dense matter characterized by $\alpha \gg \frac{p}{m}$ we get that the average value of $\omega \cos \theta$ is negative and equals

$$\langle \omega \cos \theta \rangle = -\frac{1}{3} \alpha m. \quad (29)$$

This means that in the considered case a reasonable fraction of the $SL\nu$ photons are emitted in the direction opposite to the initial neutrino momentum p , as if the neutrinos of the bunch shake off the spin light photons. It also follows, that in this case the neutrino momentum p increases as the neutrinos radiate. To illustrate this phenomena we plot in Fig.4 the $SL\nu$ radiation power spatial distribution for relativistic neutrinos with $p/m = 10$ and the density parameter equal to $\alpha = 100$. The two-dimensional cut of the spatial distribution of the radiation is shown in Fig.5.

4 $SL\nu$ polarization properties

In our previous studies [4, 5] we considered the $SL\nu$ in the low matter density limit, $\alpha \ll 1$, with account of the photon linear and circular polarizations. Here, we extend our previous consideration of the $SL\nu$ polarization properties to the case of an arbitrary matter density that enables us to treat the emitted photon polarization in the limit of very high matter density.

We first consider the two different linear photon polarizations and introduce the two orthogonal vectors

$$\mathbf{e}_1 = \frac{[\boldsymbol{\kappa} \times \mathbf{j}]}{\sqrt{1 - (\boldsymbol{\kappa} \mathbf{j})^2}}, \quad \mathbf{e}_2 = \frac{\boldsymbol{\kappa}(\boldsymbol{\kappa} \mathbf{j}) - \mathbf{j}}{\sqrt{1 - (\boldsymbol{\kappa} \mathbf{j})^2}}, \quad (30)$$

where \mathbf{j} is the unit vector pointing in the direction of the initial neutrino propagation. Decomposing the neutrino transition amplitude (7) in contributions from the photons of the two linear polarizations determined by the vectors \mathbf{e}_1 and \mathbf{e}_2 , we get

$$I^{(1),(2)} = \mu^2 \int_0^\pi \frac{\omega^4}{1 + \beta' y} \left(\frac{1}{2} S \mp \Delta S \right) \sin \theta d\theta, \quad (31)$$

where

$$\Delta S = \frac{1}{2} \frac{m^2 p \sin^2 \theta}{(E' - \alpha m)(E - \alpha m)p'}. \quad (32)$$

In the low matter density case, $\alpha \ll 1$, the total radiation power of the linearly polarized photons is

$$I^{(1),(2)} \simeq \frac{64}{3} \left(1 \mp \frac{1}{2} \right) \mu^2 \alpha^4 p^4, \quad (33)$$

in agreement with [4, 5]. Thus, the radiation powers for the two linear polarizations differ by a factor of three. Contrariwise, in all other cases the radiation powers for the two polarizations, \mathbf{e}_1 and \mathbf{e}_2 , are of the same order,

$$I^{(1)} \simeq I^{(2)} \simeq \frac{1}{2} (I^{(1)} + I^{(2)}). \quad (34)$$

It is also possible to decompose the radiation power for the circularly polarized photons. The two orthogonal vectors

$$\mathbf{e}_l = \frac{1}{\sqrt{2}} (\mathbf{e}_1 + i l \mathbf{e}_2) \quad (35)$$

describe the two photon circular polarizations ($l = \pm 1$ correspond to the right and left photon circular polarizations, respectively). For the radiation power of the circular-polarized photons we obtain

$$I^{(l)} = \mu^2 \int_0^\pi \frac{\omega^4}{1 + \beta' y} S_l \sin \theta d\theta, \quad (36)$$

where

$$S_l = \frac{1}{2} (1 + l\beta') (1 + l\beta) (1 - l \cos \theta) (1 + ly). \quad (37)$$

In the limit of low matter density, $\alpha \ll 1$, we get for the power

$$I^{(l)} \simeq \frac{64}{3} \mu^2 \alpha^4 p^4 \left(1 - l \frac{p}{2E_0} \right). \quad (38)$$

In this limiting case the radiation power of the left-polarized photons exceeds that of the right-polarized photons

$$I^{(-1)} > I^{(+1)}. \quad (39)$$

In particular, this result is also valid for non-relativistic neutrinos, $p \ll m$, for a low density with $\alpha \ll 1$.

It is remarkable that in the most interesting case of rather dense matter ($\alpha \gg \frac{m}{p}$ for $p \gg m$ and $\alpha \gg 1$ for $p \ll m$), the main contribution to the power is provided by the right-polarized photons, whereas the emission of the left-polarized photons is suppressed:

$$I^{(+1)} \simeq I, \quad (40)$$

$$I^{(-1)} \simeq 0. \quad (41)$$

Thus, we conclude that in a dense matter the $SL\nu$ photons are emitted with nearly total right-circular polarization. Note that if the density parameter changes sign, then the emitted photons will exhibit the left-circular polarization.

5 Propagation of $SL\nu$ photons in plasma

Finally, we should like to discuss in some detail restrictions on the propagation of $SL\nu$ photons, that are due to the presence of background electron plasma in the case of $p \gg m$ for the density parameter $\frac{m}{p} \ll \alpha \ll \frac{p}{m}$. Only photons with energy exceeding the plasmon frequency

$$\omega_{pl} = \sqrt{\frac{4\pi e^2}{m_e} n}, \quad (42)$$

can propagate in the plasma (here $e^2 = \alpha_{QED}$ is the fine-structure constant and m_e is the mass of the electron). From (10) and (28) it follows that the photon energy and the radiation power depend on the direction of the radiation. We can conclude that the maximal photon energy,

$$\omega_{max} = p, \quad (43)$$

and the energy of the photon emitted in the direction of the maximum radiation power,

$$\omega(\theta_{max}) = \frac{3}{4}p, \quad (44)$$

are of the same order in the case considered. For relativistic neutrinos and rather dense matter the angle θ_{max} , at which the radiation power (14) has its maximum, and the angle (43) corresponding to the maximal photon energy are both very close to zero (to illustrate this we show in Fig.6 the photon energy and radiation power angular distributions for the particular case of $m = 1 \text{ eV}$, $p = 100 \text{ MeV}$ and $n = 10^{32} \text{ cm}^{-3}$). In addition, as it follows from (26), the average photon energy $\langle \omega \rangle = \frac{1}{3}p$ is also of the order of ω_{max} and $\omega(\theta_{max})$. Therefore, the effective $SL\nu$ photon energy reasonably exceeds the plasmon frequency (42) if the following condition is fulfilled:

$$p \gg p_{min} = 3.5 \times 10^4 \left(\frac{n}{10^{30} \text{ cm}^{-3}} \right)^{1/2} \text{ eV}. \quad (45)$$

The $SL\nu$ photon emitted by a neutrino with momentum $p \gg p_{min}$ freely propagates through the plasma. For $n \sim 10^{33} \text{ cm}^{-3}$ we have $p_{min} \sim 1 \text{ MeV}$.

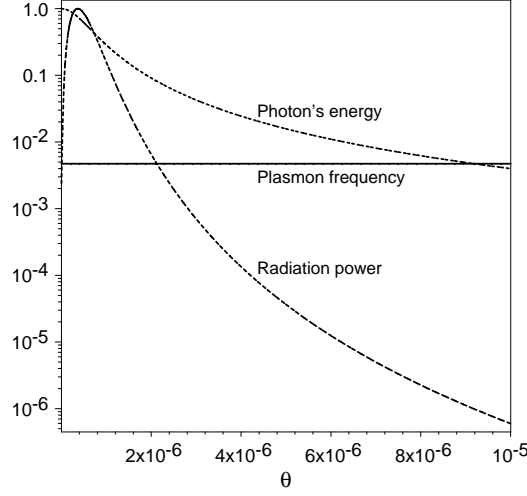


Figure 7: The angular dependence of the emitted photon energy and radiation power for the set of parameters: $m = 1 \text{ eV}, p = 100 \text{ MeV}, n = 10^{32} \text{ cm}^{-3}$. The solid line denotes the energy level that corresponds to the plasmon frequency (42).

6 Summary of $SL\nu$ properties

To conclude, we should like to mention that the obtained equation (1) is the most general equation of motion for the neutrino in which the effective potential accounts for both the charged and neutral-current interactions with the background matter. Possible effects of the motion and polarization of matter can also be incorporated [4–6].

The exact solutions obtained for the modified Dirac equation and the neutrino energy spectrum form a basis for a rather powerful method for studying different processes stimulated by neutrinos in the presence of background matter. For instance, from the neutrino energy spectrum (5) and from the matter density parameters for relativistic electron and muon neutrinos propagating in matter composed of electrons, protons and neutrons (see [4]) we can get the following expressions for the two flavour neutrinos:

$$E_{\nu_e, \nu_\mu}^{s=-1} \approx E_0 + 2\alpha_{\nu_e, \nu_\mu} m_{\nu_e, \nu_\mu}, \quad (46)$$

where

$$\alpha_{\nu_e} = \frac{1}{2\sqrt{2}} \frac{G_F}{m} \left(n_e(1 + 4\sin^2 \theta_W) + n_p(1 - 4\sin^2 \theta_W) - n_n \right), \quad (47)$$

and

$$\alpha_{\nu_\mu} = \frac{1}{2\sqrt{2}} \frac{G_F}{m} \left(n_e(4\sin^2 \theta_W - 1) + n_p(1 - 4\sin^2 \theta_W) - n_n \right), \quad (48)$$

here $n_{e,p,n}$ are the electron, proton, and neutron number densities, respectively. From the above expressions, the electron energy shift with respect to the muon energy in matter is

$$\Delta E \equiv E_{\nu_e}^{s=-1} - E_{\nu_\mu}^{s=-1} = \sqrt{2} G_F n_e. \quad (49)$$

Thus, the correct value for the neutrino energy difference corresponding to the Mikheyev-Smirnov-Wolfenstein effect [7] can be recovered.

Now, after the study of $SL\nu$ taking exactly into account matter effects, performed above, we can summarize the main features of the phenomena considered as follows:

1) a neutrino with nonzero mass and magnetic moment emits spin light when moving in dense matter,

2) in general, $SL\nu$ in matter is due to the neutrino energy dependence on the matter density and, in particular, to neutrinos of the same momentum p but of opposite helicities having different energies in matter;

3) in the particular case of electron neutrinos moving in matter composed predominantly of electrons, the matter density parameter α is positive; here the negative-helicity neutrino (the left-handed relativistic neutrino ν_L) is converted to the positive-helicity neutrino (the right-handed neutrino ν_R), giving rise to neutrino-spin polarization effect;

4) the matter density parameter α can, in general, be negative; therefore the types of initial and final neutrino states, conversion between which effectively produces the $SL\nu$ radiation, are determined by the matter composition;

5) the obtained expressions for the $SL\nu$ radiation rate and power, (13) and (14), exhibit non-trivial dependence on the density of matter and on the initial neutrino energy; in particular, as it follows from (23) and (24), in the low matter density limit the power is suppressed by an additional factor of $\frac{m}{p}$ (for $p \gg m$) or by $\frac{p}{m}$ (for $m \gg p$), in the high density limit, $\alpha \gg \frac{p}{m}$ ($p \gg m$) or $\alpha \gg \frac{m}{p}$ ($m \gg p$), the power acquires the increasing factor $\frac{p}{m}$ (for $p \gg m$) or $\frac{m}{p}$ (for $m \gg p$);

6) for a wide range of matter density parameters the $SL\nu$ radiation is beamed along the neutrino momentum p , however the actual shape of the radiation spatial distribution may vary from projector-like to cap-like, depending on the neutrino momentum-to-mass ratio and the value of α ;

7) it has been shown that for a certain choice of neutrino momentum and matter density a reasonable fraction of the emitted photons move in the direction opposite to the neutrino momentum (this interesting phenomenon arises, for instance, in the particular case of the neutrino parameter $\frac{p}{m} \sim 10$ and $\alpha \sim 100$);

8) in a wide range of matter density parameters α the $SL\nu$ radiation is characterized by total circular polarization;

9) the emitted photon energy is also essentially dependent on the neutrino energy and matter density; in particular, the photon energy increases from $\omega \sim 2p$ up to $\omega \sim \alpha m$ with the density; in the most interesting for astrophysical and cosmology applications case (when $p \gg m$ and $\frac{m}{p} \ll \alpha \ll \frac{p}{m}$) the average energy of the emitted photon is one third of the neutrino momentum p , in the case of very high density this value equals one half of the initial neutrino energy in matter.

We argue that the investigated properties of neutrino-spin light in matter may be important for experimental identification of this radiation from different astrophysical and cosmological sources. The fireball model of GRBs (see [11] for recent reviews) is one of the examples. Gamma-rays can be expected to be produced during collapses or coalescence

processes of neutron stars, owing to the $SL\nu$ mechanism in dense matter discussed. Another rather favorable situation for effective $SL\nu$ production can be realized during a neutron star being "eaten up" by the black hole at the center of our Galaxy. For estimation, let us consider a neutron star of mass $M_{NS} \sim 3M_{\odot}$ ($M_{\odot} = 2 \cdot 10^{33}g$ is the solar mass). The corresponding effective number density will be $n \sim 8 \cdot 10^{38} \text{ cm}^{-3}$ and for the matter density parameter we get $\alpha \sim 23$, if the neutrino mass is $m \sim 0.1 \text{ eV}$. For relativistic neutrino energies ($p \gg m$) the emitted $SL\nu$ photon energy, as it follows from (26), is $\langle \omega \rangle \sim 1/3p$, so that the energy range of this radiation may even extend up to energies peculiar to the spectrum of gamma-rays. Note that, as it is shown in Section 4, this radiation is characterized by the total circular polarization. This fact can be important for experimental observations.

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